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# Quantum electromagnetic wormholes and geometrical description of the electric charge

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I present and discuss a class of solutions of the Wheeler-DeWitt equation describing wormholes generated by coupling of gravity to the electromagnetic field for Kantowski-Sachs and Bianchi type-I spacetimes. Since the electric charge can be viewed as electric lines of force trapped in a finite region of spacetime, these solutions can be interpreted as the quantum corresponding to the Einstein-Rosen-Misner-Wheeler electromagnetic geon.

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## I. INTRODUCTION

Wormholes are classical or quantum solutions for the gravitational field describing a bridge between smooth regions of spacetime. In the classical case they are instantons describing a tunneling between two distant regions [1-7]; conversely, in the quantum theory wormholes are solutions of the Wheeler-DeWitt (WDW) equation [8,9] that reduce to the vacuum wave function for large three-geometries [10,11].

The existence of microscopic wormholes may have significant effects at low-energy scales (see for instance [12]), so much time has been devoted to look for particular wormhole solutions. In this paper I derive and discuss quantum wormholes generated by the electromagnetic field. Electromagnetic wormholes are very appealing for several reasons. For instance, the anisotropic nature of the electromagnetic field prevents spatially homogeneous and isotropic solutions of the field equations, so in order to describe quantum wormhole solutions we must consider minisuperspace models of dimension greater than one. As we shall see below, electromagnetic wormholes have properties that differ strongly from the properties of wormholes generated through coupling to other fields, as the scalar field. Finally, the electromagnetic field is a gauge field, and it is very important to study wormholes generated by gauge fields because the latter constitute a fundamental ingredient of the modern field theory.

I do not discuss here the most general form of the electromagnetic field, but I limit myself to a particular ansatz (for the general discussion see [13]). I consider the ansatz used in Refs. [6,7] in the context of general relativity and string theory. Hence, the quantum solutions I find here correspond to the solutions found in [6,7]. In this case the solutions of the WDW equation represent the quan-

tum analogue of a classical electromagnetic geon [14,15]. In Ref. [6] it was shown that a Lorentzian macroscopic observer looking at the wormhole sees an apparent electric charge  $Q$  even though physical charges are absent. Indeed, the electric field extends smoothly through the mouths of the wormhole so the observer in the asymptotic region interprets the electric flux as due to an apparent charge in the origin. The spacetime is described by a Reissner-Nordström type solution with mass  $M = 0$ . A Euclidean wormhole is joined at the naked singularity via a change of signature. The solutions of the WDW equation that I derive in this paper correspond to the classical solution discussed above; as we will see, the full quantum treatment avoids the problems related to the change of signature.

The outline of the paper is the following: in the next section I introduce briefly the WDW equation and the wormhole wave functions. In the third section I derive and discuss wormhole solutions of the WDW equation both for Bianchi type-I and Kantowski-Sachs models. The classical-to-quantum correspondence for the Kantowski-Sachs case is also discussed. Finally, we state our conclusions.

## II. WDW EQUATION AND WORMHOLE WAVE FUNCTIONS

Here we discuss briefly the WDW equation and the asymptotic behavior that identifies the wormhole wave functions. Let us consider a Riemannian four-dimensional space  $\Omega$  with metric  $g_{\mu\nu}$  and topology  $R \times H$ , where  $H$  is a three-dimensional compact hypersurface. The line element can be cast in the form (see for instance [14])

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (N^2 + N_i N^i) d\tau^2 + 2N_i dx^i d\tau + h_{ij} dx^i dx^j, \quad (2.1)$$

where  $N$  represents the lapse function,  $N^i$  is the shift vector and  $h_{ij}$  ( $i, j = 1, \dots, 3$ ) is the metric tensor of  $H$  ( $\tau = \text{const}$ ). The action reads

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$$S = \int_{\Omega} d^4x \sqrt{g} \left[ -\frac{M_{\text{Pl}}^2}{16\pi} {}^{(4)}R + \mathcal{L}(\phi) \right] - \frac{M_{\text{Pl}}^2}{8\pi} \int_{\partial\Omega} d^3x \sqrt{h} (\mathbf{K} - \mathbf{K}_0) . \quad (2.2)$$

Here  $M_{\text{Pl}}$  is the Planck mass,  $g = \det g_{\mu\nu}$ ,  ${}^{(4)}R$  is the curvature scalar, and  $\mathcal{L}(\phi)$  represents the Lagrangian density of a generic matter field  $\phi$ . The boundary term is required by unitarity [16];  $\mathbf{K}$  is the trace of the second fundamental form  $\mathbf{K}_{ij}$  of  $H$ , and  $\mathbf{K}_0$  is that of the asymptotic three-surface embedded in flat space. The latter contribution must be introduced if one requires the space to be asymptotically flat.

Using (2.1), (2.2) becomes

$$S = \int_{\Omega} d^4x (\pi^{ij} \dot{h}_{ij} + \pi_{\phi} \dot{\phi} - N H_0 - N^i H_i) + (\text{surface terms}) , \quad (2.3)$$

where  $\pi^{ij}$  and  $\pi_{\phi}$  are, respectively, the conjugate momenta to  $h_{ij}$  and  $\phi$ ;  $H_0$  and  $H_i$  are the Hamiltonian generators:

$$H_0 = \frac{16\pi}{M_{\text{Pl}}^2} \mathcal{H}_{ijkl} \pi^{ij} \pi^{kl} + \frac{M_{\text{Pl}}^2}{16\pi} \sqrt{h} {}^{(3)}R + \mathcal{H}_0(\phi) , \quad (2.4a)$$

$$H_i = -2\pi^j_{\phantom{j}i} \dot{h}_{ij} + \mathcal{H}_i(\phi) . \quad (2.4b)$$

Here  $\mathcal{H}_0$  and  $\mathcal{H}_i$  are the Hamiltonian generators of the matter field, and

$$\mathcal{H}_{ijkl} = \frac{1}{2\sqrt{h}} (h_{ik} h_{jl} + h_{il} h_{jk} - h_{ij} h_{kl}) \quad (2.5)$$

is the metric of the superspace.  $N$  and  $N^i$  are Lagrange multipliers that impose the constraints

$$H_0 = 0 , \quad (2.6a)$$

$$H_i = 0 . \quad (2.6b)$$

The quantization can be achieved identifying the classical quantities  $\pi^{ij}$  in (2.4) with the operators

$$\pi^{ij} \rightarrow - \left( \frac{M_{\text{Pl}}^2}{16\pi} \right) \frac{\delta}{\delta h_{ij}} \quad (2.7)$$

and analogously for the classical momenta of the matter fields. So using (2.7), (2.6a) becomes the WDW equation:

$$\left[ \mathcal{H}_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} + \sqrt{h} {}^{(3)}R + \frac{16\pi}{M_{\text{Pl}}^2} \mathcal{H}_0(\phi) \right] \Psi(h_{ij}, \phi) = 0 . \quad (2.8)$$

Here  $\mathcal{H}_0(\phi)$  is a quantum operator and  $\Psi$  represents the wave function of the system. In (2.8) there are factor ordering problems that we disregard for the moment. From (2.6b) we obtain the momentum constraint equations:

$$\left[ -2 \left( \frac{M_{\text{Pl}}^2}{16\pi} \right) \left[ \frac{\delta}{\delta h_{ij}} \right]_{|j} + \mathcal{H}_i(\phi) \right] \Psi(h_{ij}, \phi) = 0 . \quad (2.9)$$

Equations (2.9) imply that the wave function is invariant under diffeomorphisms, i.e.,  $\Psi$  is only a functional of the three-geometry and not of the particular three-metric  $h_{ij}$ .

Now, we may introduce wormhole wave functions. Following Hawking and Page [11] we define quantum wormholes as nonsingular solutions of the WDW equation and constraints (2.9) that for large three-metrics reduce to the vacuum wave function. The latter is defined by a path integral over all asymptotically Euclidean metrics  $\mathcal{C}$  (not necessarily  $R^4$ , depending on topology of the space) and matter fields vanishing at infinity:

$$\Psi(\tilde{h}_{ij}, \tilde{\phi}) = \int_{\mathcal{C}} \mathcal{D}g_{\mu\nu} \mathcal{D}\phi e^{-S[g, \phi]} . \quad (2.10)$$

Following Garay (see [10]), we may calculate the path integral (2.10). Here  $\mathcal{C}$  represents the class of three-metrics and matter fields which satisfy the conditions

$$h_{ij}(x, \tau) = \tilde{h}_{ij}, \quad \phi(x, \tau) = \tilde{\phi} , \quad (2.11a)$$

$$\phi(x, \infty) = 0, \quad h_{ij}(x, \infty) = h_{\infty} . \quad (2.11b)$$

$h_{\infty}$  is the asymptotic metric which identifies the space with minimal gravitational excitation. Evaluating (2.10) in the asymptotic limit, we obtain the asymptotic behavior of the wormhole wave function [10]

$$\Psi(\tilde{h}_{ij}) \approx \exp \left[ \int_{\tau} d^3x \pi^{ij} \tilde{h}_{ij} \right] . \quad (2.12)$$

Finally, (2.11) and (2.12) are the boundary conditions which identify the wormhole wave functions.

### III. BIANCHI TYPE-I AND KANTOWSKI-SACHS MODELS

Let us consider  $H = T^3(\chi, \theta, \varphi)$ , where  $T^3$  represents the three-torus. The line element (2.1) can be written [17,18]

$$ds^2 = \rho^2 [N^2(\tau) d\tau^2 + a^2(\tau) d\chi^2 + b^2(\tau) d\theta^2 + c^2(\tau) d\varphi^2] , \quad (3.1)$$

where  $\rho^2 = 2/M_{\text{Pl}}^2 \pi^2$  and  $\chi$ ,  $\theta$ , and  $\varphi$  are defined in the interval  $[0, 2\pi]$ .

The electromagnetic Lagrangian in (2.2) is  $\frac{1}{2} F \wedge {}^*F$ . We choose the electric field along the  $\chi$  direction [6,7]:

$$\mathbf{A} = \frac{1}{\sqrt{2\pi^3}} A(\tau) d\chi . \quad (3.2)$$

Clearly, analogous results can be obtained choosing  $\theta$  or  $\varphi$  directions. Of course, (3.2) is not the most general ansatz for the electromagnetic field (for a most general treatment, see [13]). Substituting (3.1) and (3.2) in (2.3)

and integrating over the spatial variables, the action becomes (we neglect surface terms)

$$S = \int_{\tau}^{\infty} N d\tau \left[ \frac{1}{N^2} f_{\alpha\beta} \dot{q}^{\alpha} \dot{q}^{\beta} \right], \quad (3.3)$$

where  $q = (A, a, b, c)$  and dots represent differentiation with respect to  $\tau$ .

$$f_{\alpha\beta} = \begin{pmatrix} 2bc/a & 0 & 0 & 0 \\ 0 & 0 & -c & -b \\ 0 & -c & 0 & -a \\ 0 & -b & -a & 0 \end{pmatrix} \quad (3.4)$$

is the four-dimensional metric of the minisuperspace whose line element reads

$$dS^2 = 2 \left[ \frac{bc}{a} dA^2 - cdadb - bdadc - adbdc \right]. \quad (3.5)$$

The asymptotic behavior of the wormhole wave functions (2.12) is

$$\Psi_{as}(a, b, c) \approx \exp \left[ -\frac{2}{N} \frac{d}{d\tau} (abc) \right]_{\tau}. \quad (3.6)$$

Note that the asymptotic behavior of the wormhole wave functions does not depend on the structure constants of the three-dimensional isometry group  $G$  generating a homogeneous three-surface  $H$  [17,18], so (3.6) holds for the all Bianchi and Kantowski-Sachs models. Here we will calculate (3.6) for Bianchi type-I space. The calculation for the Kantowski-Sachs space is analogous, and I will give only the result.

To calculate (3.6) we need the classical Euclidean equations of motion. Varying (3.3) with respect to the scale factor and electromagnetic potential, we obtain in the gauge  $N = 1$ :

$$\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = -\frac{\dot{A}^2}{a^2}, \quad (3.7a)$$

$$\frac{\ddot{c}}{c} + \frac{\dot{c}\dot{a}}{ca} + \frac{\ddot{a}}{a} = -\frac{\dot{A}^2}{a^2}, \quad (3.7b)$$

$$\frac{\ddot{b}}{b} + \frac{\dot{b}\dot{c}}{bc} + \frac{\ddot{c}}{c} = -\frac{\dot{A}^2}{a^2}, \quad (3.7c)$$

$$4\dot{A} \frac{bc}{a} = K, \quad (3.7d)$$

where  $K$  is the conjugate momentum to  $A$ . The Hamiltonian constraint is

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{c}\dot{a}}{ca} + \frac{\dot{b}\dot{c}}{bc} = \frac{\dot{A}^2}{a^2}. \quad (3.8)$$

Let us assume that (3.1) becomes asymptotically an Euclidean Kasner universe of indices  $p, q$ , and  $r$  when  $\tau \rightarrow \infty$  (see for instance [14]). Evaluating the curvature tensor in this limit we find that (3.1) is flat when  $p, q, r < 1$  or when  $p = 1, q = r = 0$  (and cyclic permu-

tations); in the latter case the topology is  $R^2 \times T^2$ . For these values there are no gravitational excitations in the asymptotic region. When  $p = q = r = 1$  or  $r < p = q = 1$  or  $q, r < p = 1, q, r \neq 0$  (and cyclic permutations) the Riemann tensor is nonvanishing, and then the minimal asymptotic gravitational excitation is different from zero.

The equation (3.7d) fixes the asymptotic behavior of the EM potential to be

$$A(\tau) \approx \begin{cases} \tau^{p-q-r+1}, & p-q-r \neq 1, \\ \ln(\tau), & p-q-r = 1. \end{cases} \quad (3.9)$$

Equation (2.11b) implies  $q + r > p + 1$  so the previous conditions on  $p, q$ , and  $r$  reduce to

$$(1) \quad p, q, r < 1, \quad p+1 < q+r < 2;$$

$$(2a) \quad q = 1, \quad p < r < 1;$$

$$(2b) \quad r = 1, \quad p < q < 1;$$

$$(3) \quad q = 1, \quad r = 1, \quad p < 1.$$

These relations rule out wormholes with asymptotic topology  $R^2 \times T^2$ . From the equations of motion for the scale factors (3.7) and from the Hamiltonian constraint (3.8) we easily deduce that the stress energy-momentum tensor  $T_{\mu\nu}$  of the electromagnetic field is asymptotically

$$T_{\mu\nu} \approx \tau^{-2(q+r)}. \quad (3.10)$$

The stress energy-momentum tensor must vanish for  $\tau \rightarrow \infty$  as  $G_{\mu\nu}$  or faster, so we obtain a further condition on  $q$  and  $r$ :

$$q + r \geq 1. \quad (3.11)$$

Since the electromagnetic potential along the  $\chi$  direction does not break the isotropy along  $\theta$  and  $\varphi$  directions, we can put  $b = c$  in the classical equations (3.7), (3.8) and 1-3 reduce to  $\frac{1}{2} \leq q = r \leq 1, p < 1$ . When  $q = r = \frac{1}{2}$  we must consider nonvacuum equations. Putting  $a = (\alpha\tau)^p$  and  $b = c = (\alpha\tau)^q$  and substituting in (3.7) and (3.8) we obtain

$$a = (\alpha\tau)^{-1/2}, \quad b = c = (\alpha\tau)^{1/2}, \quad (3.12a)$$

where  $\alpha \equiv \omega/2 = iK/2$ , and

$$a = (\alpha\tau)^{-1/3}, \quad b = c = (\alpha\tau)^{2/3}. \quad (3.12b)$$

In both cases the space is asymptotically flat so the minimal gravitational excitation is asymptotically zero. Choosing  $\alpha = \omega/4$  in (3.12b) and substituting  $a, b$ , and  $c$  in (3.6), we find

$$\Psi_{as} \approx e^{-\omega a(a^p b^q c^r)/2}, \quad (3.13)$$

$$\rho = \sigma + \lambda \quad \text{or} \quad \rho = 2(\sigma + \lambda) - 1,$$

where  $\sigma$  and  $\lambda$  are two arbitrary positive parameters. The wormhole wave functions can behave for large three-geometries essentially in two different ways according to the asymptotic three-metric. This occurs because the asymptotic region with minimal gravitational excitation is not unique and does not have the topology  $R^3 \times S^1$  as it happens, for instance, to the Kantowski-Sachs model. Hence, the wormhole wave functions which for large three-geometries behave as

$$\Psi_{as} \approx e^{-\omega a^2 \sqrt{bc}/2} \quad (3.14)$$

represent Riemannian spaces asymptotically of the form (3.12a) or (3.12b) and correspond to wormholes joining two asymptotically flat regions with metric (3.12a) and (3.12b).

Now we are able to find wormhole solutions for the Bianchi type-I model. Choosing the Hawking-Page prescription for the factor order [11], the WDW equation (2.8) can be cast in the form

$$\Delta \Psi(a, b, c, A) = 0, \quad (3.15)$$

where  $\Delta$  is the Laplace-Beltrami covariant operator in the minisuperspace:

$$\begin{aligned} \Delta = & \frac{2}{c} \partial_a \partial_b + \frac{2}{b} \partial_a \partial_c + \frac{2}{a} \partial_b \partial_c - \frac{a}{bc} \partial_a^2 - \frac{b}{ac} \partial_b^2 - \frac{c}{ab} \partial_c^2 \\ & + \frac{1}{bc} \partial_a - \frac{1}{ac} \partial_b - \frac{1}{ab} \partial_c - \frac{a}{bc} \partial_A^2. \end{aligned} \quad (3.16)$$

A set of solutions of (3.15) as

$$\begin{aligned} \Psi(a, b, c, A; p, k, \omega) = & \frac{1}{\sqrt{bc}} a^{i(p+k)/2} b^{ip/2} c^{ik/2} \\ & \times K_{i\sqrt{pk}}(\omega a) e^{i\omega A}, \end{aligned} \quad (3.17)$$

where  $p$ ,  $k$ , and  $\omega$  are real constants, and  $K$  is the modified Bessel function. The wave functions (3.17) oscillate an infinite number of times approaching the origin. They are damped for large values of  $a$ , but not for large  $b$  and  $c$  because they oscillate in the  $b$ - $c$  plane. This feature is not surprising, since we have chosen a electromagnetic field that “lives” in the one-sphere with radius  $a$ , so its dynamics does not depend on  $b$  and  $c$  [19]. Using the variables  $\ln b$  and  $\ln c$ , we see that the oscillating factors  $b^{ip/2}$  and  $c^{ik/2}$  look formally as  $e^{i\omega A}$ . So, as far as the dynamics of the wormhole is concerned, the scale factors  $b$  and  $c$  behave essentially as matter fields and together with  $A$  determine the extent of the wormhole mouth [19]. Note that for real values of  $\omega$  there is a real flux of the electric field through any closed  $T^2(\theta, \phi)$  surface due to the electric field along  $\chi$ . The physical meaning of this flux will be more clear in a moment. Finally, the factor  $1/\sqrt{bc}$  is eliminated by the measure in the integral when we deal with matrix elements, so the probability density  $\sqrt{f}|\Psi|^2$  is finite everywhere.

Considering a linear combination of wave functions (3.17), we can find regular wormhole wave functions. Let us put  $p = \xi^2 k$ , where  $\xi$  is a real number, and take the Fourier transform [20]. We obtain (see [21], for instance):

$$\begin{aligned} \Psi(a, b, c, A; \mu, \xi, \omega) &= \int dk e^{i\mu k} \Psi(a, b, c, A; k, \xi, \omega) \\ &= \frac{1}{\sqrt{bc}} e^{i\omega A} e^{-\omega a \cosh[\ln(a^\sigma + \lambda b^\sigma c^\lambda) + \mu]}, \end{aligned} \quad (3.18)$$

where  $\sigma = \xi/2$  and  $\lambda = 1/2\xi$ . Wave functions (3.18) represent Riemannian spaces asymptotically of the form (3.12a) and are the analogue for the electromagnetic field of the solutions found by Campbell and Garay in Ref. [22] for a scalar field in Kantowski-Sachs space. Solutions (3.18) are again eigenfunctions of the operator  $\partial/\partial A$ , so there is a real flux through any closed  $T^2(\theta, \phi)$  surface as for (3.17). The physical interpretation of (3.18) must take some care since the regularity at small three-geometries means that the space closes regularly there, so the electromagnetic flux cannot go through the wormhole throat. We will see below the physical meaning of this feature.

Choosing  $\xi = 1$  we obtain

$$\Psi(a, b, c, \mu, \omega) = \frac{1}{\sqrt{bc}} e^{i\omega A} e^{-\omega a \cosh[\ln(a\sqrt{bc}) + \mu]}. \quad (3.19)$$

Now, let us put for simplicity  $\mu = 0$ . Equation (3.19) can be cast in the form

$$\Psi(a, b, c, A; \omega) = \frac{1}{\sqrt{bc}} e^{-\omega a^2 \sqrt{bc}/2} e^{-\omega/2\sqrt{bc}} e^{i\omega A}, \quad (3.20)$$

which coincides with a Kontorovich-Lebedev transform (see [21]) of (3.17) with respect to the index  $k = p$ . Using a different type of Kontorovich-Lebedev transform we can find a further solution

$$\begin{aligned} \Psi(a, b, c, A; \omega) = & \frac{1}{\sqrt{bc}} \left( a^2 \sqrt{bc} - \frac{1}{\sqrt{bc}} \right) e^{-\omega a^2 \sqrt{bc}/2} \\ & \times e^{-\omega/2\sqrt{bc}} e^{i\omega A}. \end{aligned} \quad (3.21)$$

The asymptotic behaviors of (3.20) and (3.21) suggest us the use in the WDW equation of the new variables

$$\eta^2 = \omega a^2 \sqrt{bc}/2, \quad \xi^2 = \omega/2\sqrt{bc}. \quad (3.22)$$

Recalling (3.15) we obtain

$$\Psi_n(\eta, \xi, A; \omega) = \psi_n(\eta + \xi) \psi_n(\eta - \xi) \xi^2 e^{i\omega A}, \quad (3.23)$$

where  $\psi_n(x)$  is the harmonic wave function of order  $n$ :

$$\psi_n(x) = \frac{1}{(2^n n! \sqrt{\pi})^{1/2}} H_n(x) e^{-x^2/2}. \quad (3.24)$$

Solutions (3.20) and (3.21) correspond (apart from normalization factors) to  $\Psi_0$  and  $\Psi_1$ . As in the previous set of solutions, we have a nonzero flux even if the wave functions are regular for small three-geometries. This important property makes (3.19) and (3.23) very different from other quantum wormholes known in literature. As we have seen, *both for singular and regular wave functions* there is a real flux. For solutions (3.17) this is not surprising, because one can imagine the flux coming out or going into the singularity at the origin. However, for

solutions (3.23) where can the flux go?

To answer this question and shed light on the physical interpretation of the solutions, we have to consider the structure of the (Euclidean) electromagnetic field [23]. As we have said before, the ansatz (3.2) represents a purely electric field along the  $\chi$  direction, i.e., a electromagnetic field whose dynamics is confined in the one-sphere with radius  $a$ . Conversely, the asymptotic behavior of the wave function for large three-geometries and the throat depend on  $b$  and  $c$ . Hence, the dynamics of the electromagnetic field is decoupled from the dynamics of the wormhole and the flux of the electric field must coincide for regular and nonregular wave functions [19].

Even though there are no physical charges in the field equations, the observer in the asymptotically flat region measures a real finite flux and sees an *apparent* charge in the origin. Thus the geometry must be nontrivial. Since the solutions (3.23) describe asymptotically flat spaces, the electromagnetic field is confined in a finite region. We can conclude that (3.23) describe the quantum analogue of a electromagnetic geon because the charge can be seen as an electric field trapped in a finite region of space, without any source.

This interpretation holds also for the Kantowski-Sachs model. Moreover, in the latter case we can compare the quantum results to the classical ones, since the classical wormhole solution corresponding to the ansatz (3.2) is known. Let us now discuss briefly the Kantowski-Sachs model. The quantum wormhole solutions for the ansatz (3.2) have been found in [24]. They are

$$\Psi(a, b, A; \nu, \omega) = K_{i\nu}(\omega a) K_{i\nu}(4ab) e^{\pm i\omega A}. \quad (3.25)$$

The wave functions (3.25) are singular in the origin, where they oscillate an infinite number of times. For  $4ab < |\nu|$  the wave functions oscillate (Lorentzian region), conversely for  $4ab > |\nu|$  they are exponentially damped (Euclidean region). The asymptotic behavior corresponds to the flat  $R^3 \times S^1$  space. Hence, (3.25) represent quantum wormholes with throat  $4ab = |\nu|$  joining two asymptotically flat regions with topology  $R^3 \times S^1$ . Note that the size of the throat does not depend on the scale factor  $a$  nor on the parameter  $\omega$ .

Multiplying (3.25) by a factor  $\nu \tanh(\pi\nu)$  and taking the Kontorovich-Lebedev transform, (see [21]) we obtain:

$$\Psi(a, b, A; \omega) = \frac{2\sqrt{b\omega}}{\omega + 4b} e^{-a(\omega+4b)} e^{\pm i\omega A}. \quad (3.26)$$

We can easily verify that (3.26) is regular everywhere and its asymptotic behavior for  $b \rightarrow \infty$  coincides with the behavior of (3.25). As in Bianchi type-I model, we have again regular solutions but a nonzero electromagnetic flux, so (3.26) must describe a nontrivial topology. Indeed, since there are no physical sources for the electromagnetic field, the Gauss law should imply a vanishing flux through closed surfaces around the origin. We conclude that (3.26) represent a quantum electromagnetic geon. The latter interpretation is supported by the classical solution corresponding to this model. As shown in [6]

for the classical Einstein-Maxwell theory, a macroscopic observer measures an apparent electric charge  $Q$  even though physical charges are absent. Since the spacetime is not trivial, the electric field extends smoothly beyond an Euclidean wormhole throat joined to the Lorentzian isometric regions via a classical change of signature. The Lorentzian geometry is described by the metric:

$$ds^2 = - \left(1 + \frac{Q^2}{R^2}\right) dT^2 + \left(1 + \frac{Q^2}{R^2}\right)^{-1} dR^2 + R^2 d\Omega_2^2, \quad (3.27)$$

where the radial coordinate  $R$  ranges in  $]0, \infty[$ . The line element (3.27) is of a Reissner-Nordström type and the joining occurs in the naked singularity at  $R = 0$ . Indeed, the latter can be “expanded” and continued analytically in the Euclidean space, where the solution is everywhere regular. This Euclidean region describes the wormhole. The wave functions (3.26) are the quantum analogue of this picture. In the full quantum treatment we avoid the problems of the classical case, namely the “ad hoc” change of signature: solutions (3.26) are regular everywhere and thus describe a geometry that closes regularly. In the classical theory, the lines of force of the electric field are convergent at  $R = 0$ , where the Lorentzian spacetime becomes singular. So we need a little “trick,” namely the transition from a Lorentzian to an Euclidean region. In the quantum theory, we have solutions regular everywhere and no tricks are necessary. We stress that these results depend strongly on the ansatz (3.2) chosen for the electromagnetic field. Naturally, identical conclusions can be drawn for the Bianchi type-I model.

#### IV. CONCLUSIONS

In this paper we have found and discussed a class of solutions of the WDW equation. These solutions represent wormholes generated by coupling gravity and the electromagnetic field. They have topology  $R \times T^3$  or  $R \times S^1 \times S^2$ . We have considered a particular ansatz for the electromagnetic field, namely a purely electric field along a  $S^1$  section. These solutions can be interpreted as the quantum corresponding of a electromagnetic geon. The electric charge is viewed as electric lines of force trapped in a finite region of spacetime. This interpretation has been discussed in detail for the  $R \times S^1 \times S^2$  topology, and the correspondence with the classical Einstein-Maxwell theory has been explored.

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- [1] S. B. Giddings and A. Strominger, Nucl. Phys. **B306**, 890 (1988).
- [2] J. J. Halliwell and R. Laflamme, Class. Quantum Grav. **6**, 1839 (1989).
- [3] D. H. Coule and K. Maeda, Class. Quantum Grav. **7**, 955 (1990).
- [4] S. W. Hawking, Phys. Lett. B **195**, 337 (1987).
- [5] A. Hosoya and W. Ogura, Phys. Lett. B **225**, 117 (1989).
- [6] M. Cavaglià, V. de Alfaro, and F. de Felice, Phys. Rev. D **49**, 6493 (1994).
- [7] M. Cadoni and M. Cavaglià, Phys. Rev. D (to be published).
- [8] B. DeWitt, Phys. Rev. **160**, 1113 (1967).
- [9] J. A. Wheeler, in *Battelle Rencontres: 1967 Lectures in Mathematics and Physics*, edited by C. DeWitt and J. A. Wheeler (Benjamin, New York, 1968).
- [10] L. J. Garay, Phys. Rev. D **44**, 1059 (1991).
- [11] S. W. Hawking and D. N. Page, Phys. Rev. D **42**, 2655 (1990).
- [12] S. Coleman, Nucl. Phys. **B310**, 643 (1988).
- [13] H. F. Dowker, Nucl. Phys. **B331**, 194 (1990).
- [14] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, New York, 1973).
- [15] C. W. Misner and J. A. Wheeler, Ann. Phys. (N.Y.) **2**, 525 (1957).
- [16] S. W. Hawking, in *General Relativity, an Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).
- [17] M. P. Ryan and L. C. Shepley, *Homogeneous Relativistic Cosmologies* (Princeton University Press, Princeton, NJ, 1975).
- [18] R. T. Jantzen, in *Cosmology of the Early Universe*, edited by L. Z. Fang and R. Ruffini (World Scientific, Singapore, 1984).
- [19] We are very grateful to L. J. Garay for suggesting this interpretation.
- [20] For the physical meaning of the Fourier transform in minisuperspace models, see, for instance, L. J. Garay, Phys. Rev. D **48**, 1710 (1993); G. A. Mena Marugán, "Bases of Wormholes in Quantum Cosmology," Report No. CGPG-94/4-4, gr-qc/9404042 (unpublished); Phys. Rev. D **50**, 3923 (1994).
- [21] *Tables of Integral Transforms* (Bateman Manuscript Project), edited by A. Erdélyi *et al.* (McGraw-Hill, New York, 1954), Vols. I and II.
- [22] L. M. Campbell and L. J. Garay, Phys. Lett. B **254**, 49 (1991).
- [23] For the classical Einstein-Maxwell theory in the Euclidean space, see, for instance, D. Brill, in *Proceedings of Louis Witten Festschrift* (World Scientific, Singapore, in press).
- [24] M. Cavaglià, Mod. Phys. Lett. A **9**, 1897 (1994).